ESCSI TECHNICAL INFORMATION SHEET
EXPANDED SHALE, CLAY AND SLATE INSTITUTE / Rotary Kiln Structural lightweight Aggregate

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Some answers to the concrete designer's often-asked question…

What About Deflection?

Since World War II, a new trend has been born in the structural design of reinforced concrete. Its genesis lies in all the influences, which may be summed up in that emphatic word "cost," and its growth in the more or less intangible thing called the "new look." Monotony in column spacing and routine layouts has given way to set-back columns, slender members with long spans, provision either full or partial against the pressures of blasts and other contemporary practices. The 20th century shifted the engineering design paradigm to performance-based design, highlighting the ductility of reinforced concrete structures and requiring special detailing to accommodate the target displacement of buildings. Infrastructure resilience has also emphasized the importance of structural deformations at service and extreme loadings.

Structural members-beams, slabs, and columns have been on a rigorous diet and have lost girth. We have been reducing and have become more flexible. We deflect more easily. Engineers, architects, and owners have become deflection conscious. They cannot see the stress in an overhanging canopy; instead, they see a deflection, and some are alarmed at what they see.

Consequently, the designer must also become deflection conscious for no reason other than to allay needless fear. The reinforced concrete design relies on assumptions and empirical formulas proven by tests and observed performance. Deflection formulae also rely on assumptions. Because of the nature of past designs, we have not had too much experience in deflection computations in concrete. The fundamental theory is sound, and the investigation is not too complex.

Among uncertainties inherent in the computation of deflections of reinforced concrete beams and slabs are the values of E (modulus of elasticity) and I (the moment of inertia). The formation of cracks and the shift from an uncracked to a cracked section considerably influence these values. The deflection also depends upon the value of n, called the modular ratio. Specifically, n is the quotient obtained by dividing the modulus of elasticity of steel by the modulus of elasticity of concrete, or E_s/E_c.

E_c, in turn, is a variable subject to much uncertainty. Under long-continued stress, it decreases, and we say that plastic flow has set in. Therefore, the value of n, which was assumed to be 10 for design purposes for 3000 psi concrete, may rise to many times 10 under continued heavy loading. Hence, the n value is likely between 5 and 7 before loading.

The computations for deflections are further complicated because we deal with a finite visual quantity and not an abstract stress value. We can see and measure the deflection. In the design of a beam or slab, for example, under the ACI Code, we give the concrete no credit in tension. In compression, the concrete strain varies from a maximum at the top fiber (or bottom fiber for a negative moment) to zero at the neutral axis. This assumption does not consider the shear effect, and therefore, it is far from accurate, but the result is the selection of dimensions and reinforcement, which are safe and commercially competitive.

Further, the nonlinear stress-strain relationship of concrete before and after the peak stress adds to the complication of deflection calculations. Various attempts have been (and are still being) made to enhance the constitutive stress-strain modeling of concrete to account for the elastoplastic nonlinear behavior of concrete, the combination of axial, flexural, and shear stresses, the interaction between concrete and reinforcing members, including longitudinal, transverse, and fiber reinforcements, and the cyclic behavior of reinforced concrete sections.

When dealing with deflections, however, we are after an absolute value and not a comparative value. Another aspect of the problem is that deflections of concrete members are small under working loads. Perhaps the preceding comments may be more easily put in three dimensions by illustrating them with an example. This stage introduces one more consideration by assuming lightweight aggregates application in some cases. Lightweight aggregates further permit flexibility because a member in normal weight concrete weighs 50 percent more than the same member in lightweight concrete and the selection of equipment for handling and erection hangs on the weight of the member. Precast concrete is also coming more to the fore daily with lightweight applications. One more forward step is using prestressed concrete, which may or may not be precast but can benefit from the low density of lightweight concrete.

The presented example refers to Sections 7.7 and 13.11 Examples of the Reinforced Concrete Design Handbook: A Companion to ACI 318-19.

Let's assume a 36-foot span interior continuous reinforced concrete beam, built integrally with a 7 in. slab within a six-bay moment frame. The beam width is 18 in., and column dimensions are 24 in. by 24 in. The beam carries an additional 15 psf service dead load and 65 psf service live load over a 14-ft tributary width. The specified compressive strength of concrete is 5000 psi, and the reinforcing steel yield stress is 60,000 psi. The equilibrium density of the lightweight concrete is 120 pcf. Hence, the modification factor, λ per Table 19.2.4.1(a) is 0.9.

Use ACI 318-19 and refer to the SP-17 Volume 1: Member Design and the MNL-17 Volume 3: Design Aids of The ACI Reinforced Concrete Design Handbook: A Companion to ACI 318-19.

The depth of the beam per Table 9.3.1.1 and modification in 9.3.1.1.2 is:

$$h = \frac{\ell}{18.5}(1.09) = \frac{36 \times 12}{18.5}(1.09) = 25.45\ in.$$

Use 30 in. The minimum depth for normal weight concrete is 23.34 in., which would not change the choice of the practical depth of the beam.

Self-weight of beams and slabs are:

$$w_b = [(18/12)(30/12)](120/1000) = 0.450\, kip/ft$$

$$w_s = [(14 \times 12 - 18)/12(7/12)](120/1000) = 0.875\, kip/ft$$

The ultimate load is:

$$w_u = 1.2[0.450 + 0.875 + (15 \times 14/1000)] + 1.6[(65 \times 14/1000)] = 3.298\, kip/ft$$

The self-weight reduction due to the application of lightweight concrete is 20%. The reduction in ultimate load is 10.76%.

The effective width of the slab on each side of the web from Table 6.3.2.1 is:

$$min \begin{cases} 8h_{slab} = 8 \times 7 = 56\ in. \\ S_w/2 = 14 \times \frac{12}{2} = 84\ in. \\ \ell_n/8 = \frac{[(36 \times 12) - 24]}{8} = 51\ in. \text{ Controls} \end{cases}$$

Hence, the flange width is:

$$b_f = \ell_n/8 + b_f + \ell_n/8 = 51 + 18 + 51 = 120\ in.$$

Approximate moments and shear for an interior beam following section 6.5 are:

$$M_{uEe}^{-} = \frac{w_u {\ell_n}^2}{16} = \frac{3.298 \times 34^2}{16} = 238.28\ ft \cdot kip \text{ for the interior face of exterior support}$$

$$M_{uEi}^{-} = \frac{w_u {\ell_n}^2}{10} = \frac{3.298 \times 34^2}{10} = 381.25\ ft \cdot kip \text{ for the exterior face of the first interior support}$$

$$M_{uE}^{+} = \frac{w_u {\ell_n}^2}{14} = \frac{3.298 \times 34^2}{14} = 272.32\ ft \cdot kip \text{ for end span}$$

$$M_{uI}^{-} = \frac{w_u {\ell_n}^2}{11} = \frac{3.298 \times 34^2}{11} = 346.59\ ft \cdot kip \text{ for the face of all other supports}$$

$$M_{uI}^{+} = \frac{w_u {\ell_n}^2}{16} = \frac{3.298 \times 34^2}{16} = 238.28\ ft \cdot kip \text{ for interior spans}$$

$$V_{uE} = 1.15 \frac{w_u \ell_n}{2} = \frac{3.298 \times 34}{2} = 64.48\ ft \cdot kip \text{ for the exterior face of the first interior support}$$

$$V_{uI} = \frac{w_u \ell_n}{2} = \frac{3.298 \times 34}{2} = 56.07\ kip \text{ for the face of all other supports}$$

The effective depth of the beam, assuming No. 3 stirrups and No. 7 longitudinal bars, and 1.5 in. cover is:

$$d = h - cover - d_{tie} - \frac{d_b}{2} = 30 - 1.5 - 0.375 - \frac{0.875}{2} = 27.6\ in.$$

Let's use 27.5 inches.

Per Table 22.2.2.4.3:

$$\beta_1 = 0.85 - \frac{0.05(f'_c - 4000)}{1000} = 0.85 - \frac{0.05(5000 - 4000)}{1000} = 0.80$$

The equivalent concrete compressive depth *a* is:

$$C = T$$

$$0.85 f'_c ba = A_s f_y$$

The beam width differs for positive and negative moments:

$$b^+ = b_f = 120\ in.$$

$$b^- = b_w = 18\ in.$$

Thus:

$$a^+ = \frac{A_s 60{,}000}{0.85 \times 5000 \times 120} = 0.118 A_s$$

$$a^- = \frac{A_s 60{,}000}{0.85 \times 5000 \times 18} = 0.784 A_s$$

Required reinforcement for the maximum flexural moments obtained from the approximate method above follow this equation:

$$M_u \leq \emptyset M_n = 0.9 A_s f_y \left(d - \frac{a}{2}\right)$$

Hence, steel reinforcement follows:

$${A_{sEe}}^- = 1.981\ in.^2 \text{ Use 4\#7 or 2.4 } in.^2$$

$${A_{sEi}}^- = 3.229\ in.^2 \text{ Use 6\#7 or 3.6 } in.^2$$

$${A_{sE}}^+ = 2.211\ in.^2 \text{ Use 4\#7 or 2.4 } in.^2$$

$${A_{sI}}^- = 2.922\ in.^2 \text{ Use 5\#7 or 3.0 } in.^2$$

$${A_{sI}}^+ = 1.934\ in.^2 \text{ Use 4\#7 or 2.4 } in.^2$$

The normalweight concrete requires 12% more reinforcement, which is an additional #7 for each section except the exterior face of the exterior beam and the mid-span of the interior beam. The additional #7 increases the actual reinforcement by nearly 20% in this example.

Hence, tension strain in reinforcement follows these equations:

$$c = a/\beta_1$$

$$\varepsilon_t = \frac{\varepsilon_{cu}}{c}(d - c) = \frac{0.003}{a/0.85}\left(27.5 - \frac{a}{0.85}\right)$$

$$\varepsilon_{tEe}{}^{-} = 0.0320$$

$$\varepsilon_{tEi}{}^{-} = 0.0203$$

$$\varepsilon_{tE}{}^{+} = 0.2295$$

$$\varepsilon_{tI}{}^{-} = 0.0250$$

$$\varepsilon_{tI}{}^{+} = 0.2295$$

The following table summarizes the analysis and design parameters.

Location	M_u	a/A_s	A_{sreq}	No. #7	A_s	a	c	ε
The interior face of the exterior support	238.28	0.784	1.99	4	2.4	1.886	2.357	0.0320
The exterior face of the first interior support	381.25	0.784	3.23	6	3.6	2.829	3.536	0.0203
Mid end span	272.32	0.118	2.21	4	2.4	0.284	0.355	0.2295
The face of all other supports	346.59	0.784	2.92	5	3.0	2.357	2.946	0.0250
Mid interior spans	238.28	0.118	1.93	4	2.4	0.284	0.355	0.2295

The minimum strain value for tension-controlled design per Table 21.2.2 is:

$$\varepsilon_{ty} + \varepsilon_{cu} = 0.002 + 0.003 = 0.005$$

All values exceed 0.005; therefore, tension controls, that is $\phi = 0.9$.

Minimum reinforcement follows Section 9.6.1:

$$A_{smin} = max \begin{cases} \frac{3\sqrt{f_c'}}{f_y} b_w d = \frac{3\sqrt{5000}}{60{,}000} 18 \times 27.5 = 1.75\ in.^2 \text{ controls} \\ \frac{200}{f_y} b_w d = \frac{200}{60{,}000} 18 \times 27.5 = 1.65\ in.^2 \end{cases}$$

The minimum reinforcement at every section along the length of the beam is 3#7.

The shear demand at the critical section at a distance *d* from the face of the support is:

$$V_{u@d} = V_u - w_u d = 64.48 - 3.298 \times \frac{27.5}{12} = 56.92 kip$$

The shear strength of concrete is:

$$V_c = 2\lambda\sqrt{f_c'} b_w d = 2(0.9)\sqrt{5000} 18 \times \frac{27.5}{1000} = 63.0\ kip$$

$$\emptyset V_c = 0.75 \times 63 = 47.25\ kip < V_{u@d} = 56.92\ kip \text{ NG}$$

Therefore, shear reinforcement is required.

The section meets the cross-sectional dimension per Eq. 22.5.1.2:

$$V_u = 56.92 \le \emptyset\left(V_c + 8\sqrt{f_c'} b_w d\right) = 0.75\left(63 + \frac{8\sqrt{5000} \times 18 \times 27.5}{1000}\right) = 257 kip \text{ OK}$$

Transverse shear reinforcement per Eq. 22.5.10.1 is:

$$V_s \ge \frac{V_u}{\emptyset} - V_c = \frac{56.92}{0.75} - 47.25 = 28.64$$

The threshold value for the shear reinforcement strength is:

$$V_s = 28.64 \le 4\lambda\sqrt{f_c'} b_w d = \frac{4(0.9)\sqrt{5000} \times 18 \times 27.5}{1000} = 126.0\ kip \text{ OK}$$

Using No. 3 stirrups:

$$V_s = \frac{A_v f_{yt} d}{s}$$

$$s = \frac{2 \times 0.11 \times 60{,}000 \times 27.5}{28.64 \times 1000} = 12.66\ in.$$

The spacing for normal weight concrete would be 15.5in., which is not controlling the design due to maximum stirrup spacing criteria.

The maximum stirrup spacing is:

$$s_{max} = min\begin{cases} \frac{d}{2} = \frac{27.5}{2} = 13.8\ in. \text{ controls} \\ 24\ in. \end{cases}$$

Use 12 in. spacing.

Specified minimum shear reinforcement is:

$$A_{v,min} = \begin{cases} 0.75\sqrt{f_c'}\frac{b_w}{f_{yt}}s = 0.75\sqrt{5000}\frac{18}{60{,}000}12 = 0.192\ \ in.^2 \text{ OK} \\ 50\frac{b_w}{f_{yt}}s = 50\frac{18}{60{,}000}12 = 0.180\ in.^2 \text{ OK} \end{cases}$$

The elastic deflection of a continuous beam depends on the loading pattern along the multiple bay moment frame. For simplicity of this example, let's consider the case of a single beam with fixed supports simulating uniform loading on all bays. The following equation is available through a first-order elastic analysis for homogenous materials and instant deflection of a fixed-fixed beam:

$$\Delta = \frac{w\ell^4}{384E_cI_e}$$

To use this formula on a concrete beam, we must approximate a realistic value of *E* in the long-term, say 2 to 5 years, when plastic flow shall have ceased. Further, the value of *I* requires calculation based on cracked concrete.

The modulus of elasticity per Eq. 19.2.2.1.a are:

$$E_c = {w_c}^{1.5}33\sqrt{f_c'} = 120^{1.5}33\sqrt{5000}/1000 = 3{,}067\ ksi$$

Hence.

$$n = \frac{E_s}{E_c} = \frac{29{,}000}{3{,}067} = 9.5$$

The service load is:

$$w_a = [0.450 + 0.875 + (15 \times 14/1000)] + [(65 \times 14/1000)] = 2.445\ kip/ft$$

The load reduction due to the application of lightweight aggregate is 9.8%.

Moments at service load are:

$$M_{aI}{}^{-} = \frac{w_u\ell^2}{12} = \frac{2.445 \times 36^2}{12} = 264.06\ ft \cdot kip$$

$$M_{aI}{}^{+} = \frac{w_u\ell^2}{24} = \frac{2.445 \times 36^2}{24} = 132.03\ ft \cdot kip$$

The rupture stress per ACI 318 Eq. 19.2.3.1 is:

$$f_r = 7.5\lambda\sqrt{f'_c} = 7.5 \times 0.75 \times \sqrt{5000} = 397.75psi$$

Gross moment of inertia using Design Aid, Deflection 13.5 is:

$$\alpha_b = \frac{b}{b_w} = \frac{120}{18} = 6.667$$

$$\beta_h = \frac{h_f}{h} = \frac{7}{30} = 0.233$$

$$K_{i4} = 1 + (\alpha_b - 1)\beta_h^3 + \frac{3(1-\beta_h)^2\beta_h(\alpha_b - 1)}{1 + \beta_h(\alpha_b - 1)} = 2.076$$

$$I_g = \frac{K_{i4}b_w h^3}{12} = 2.076 \times 18 \times \frac{30^3}{12} = 84{,}076.66in.^4$$

Similarly, section properties follow:

$$y_t{}^+ = \frac{b_w h\frac{h}{2} + (b_f - b_w)t_s\left(h - \frac{t_s}{2}\right)}{b_w h + (b_f - b_w)t_s} = \frac{18 \times 30\frac{30}{2} + (120 - 18)7\left(30 - \frac{7}{2}\right)}{18 \times 30 + (120 - 18)7} = 21.55\ in.$$

$$y_t{}^- = h - y_t{}^- = 30 - 21.55 = 8.45\ in.$$

$$I_g = \frac{b_w h^3}{12} + b_w h\left(\frac{h}{2} - y_t\right)^2 + \frac{(b_f - b_w)t_s{}^3}{12} + (b_f - b_w)t_s\left(h - \frac{t_s}{2} - y_t\right)^2$$
$$= \frac{18 \times 30^3}{12} + 18 \times 30\left(\frac{30}{2} - 21.55\right)^2 + \frac{(120-18)7^3}{12}$$
$$+ (120 - 18)7\left(30 - \frac{7}{2} - 21.55\right)^2 = 84{,}077.64\ in.^4$$

The cracking moment for T-section using Design Aid, Deflection 13.1 is:

$$K_{cr} = \frac{f_r}{12{,}000}\frac{h^2}{6} = \frac{7.5\sqrt{5000}}{12{,}000}\frac{30^2}{6} = 6.629ft \cdot kip$$

$$\alpha_b = \frac{b}{b_w} = \frac{120}{18} = 6.667$$

$$\beta_h = \frac{h_f}{h} = \frac{7}{30} = 0.233$$

$$K_{crt}{}^+ = \frac{1 + (\alpha_b - 1)\beta_h\left[4 - 6\beta_h + 4\beta_h^2 + (\alpha_b - 1)\beta_h^3\right]}{1 + (\alpha_b - 1)\beta_h(2 - \beta_h)} = 1.445$$

$$K_{crt}{}^- = \frac{1 + (\alpha_b - 1)\beta_h\left[4 - 6\beta_h + 4\beta_h^2 + (\alpha_b - 1)\beta_h^3\right]}{1 + \beta_h^2(\alpha_b - 1)} = 3.684$$

$$M_{cr}{}^{+} = b_w K_{cr} K_{crt}(0.75) = 18 \times 6.629 \times 1.445 \times 0.75 = 129.3 ft \cdot kip$$
$$M_{cr}{}^{-} = b_w K_{cr} K_{crt}(0.75) = 18 \times 6.629 \times 3.684 \times 0.75 = 329.7 ft \cdot kip$$

Cracking moments per Eq., 24.2.3.5b are:

$$M_{cr}{}^{+} = \frac{f_r I_g}{y_t} = \frac{397.75 \ \times 84{,}077.64}{21.55} \times \frac{1}{12 \times 1000} = 129.32\ ft \cdot kip$$
$$M_{cr}{}^{-} = \frac{f_r I_g}{y_t} = \frac{397.75 \ \times 84{,}077.64}{8.45} \times \frac{1}{12 \times 1000} = 329.80\ ft \cdot kip$$

Following Table 24.2.3.5:

$$M_{aI}{}^{+} = 132.03\ ft \cdot kip > \frac{2}{3} M_{cr}{}^{+} = \frac{2}{3} 129.32 = 86.21\ ft \cdot kip$$
$$M_{aI}{}^{-} = 264.06\ ft \cdot kip > \frac{2}{3} M_{cr}{}^{-} = \frac{2}{3} 329.80 = 219.87\ ft \cdot kip$$

Cracked-section moment of inertia following Design Aid, Deflection 13.6.1 is:

$$\rho^{-} = \frac{A_s}{b_w d} = \frac{5 \times 0.6}{18 \times 27.5} = 0.0061$$
$$\rho^{+} = \frac{A_s}{b_w d} = \frac{4 \times 0.6}{18 \times 27.5} = 0.0048$$
$$\beta_c = \left(\frac{b}{b_w} - 1\right)\frac{h_f}{d \rho_w n} = \left(\frac{120}{18} - 1\right)\frac{7}{27.5 \times 0.0048 \times 9.5} = 31.632$$
$$a = 0.118 A_s = 0.118 \times 2.4 = 0.283$$
$$c = \frac{a}{\beta_1} = \frac{0.283}{0.80} = 0.354$$
$$a' = 0.784 A'_s = 0.784 \times 3.0 = 2.352$$
$$c' = \frac{a'}{\beta_1} = \frac{2.352}{0.80} = 2.94$$
$$d' = h - d = 30 - 27.5 = 2.5 in.$$
$$K_{i2} = \left[\frac{(c/d)^3}{3} + \rho n\{1 - (2\,c/d) + (c/d)^2\} + \rho n \beta_c \left\{(c/d)^2 - (2\,c/d)\frac{d'}{d} + \left(\frac{d'}{d}\right)^2\right\}\right] = 0.0532$$
$$I_{cr}{}^{+} = K_{i2} b_w d^3 = 0.0532 \times 18 \times 27.5^3 = 19{,}921.88 in.^4$$

The effective moment of inertia using Design Aid, Deflection 13.7.1 is:

$$\frac{M_{cr}}{M_a}=\frac{129.32}{132.03}=0.9795$$

$$\frac{I_{cr}}{I_g}=\frac{19{,}921.88}{84{,}077.64}=0.2369$$

$$I_e{}^{+}=\frac{I_{cr}}{1-\left(\frac{\frac{2}{3}M_{cr}}{M_a}\right)^2\left(1-\frac{I_{cr}}{I_g}\right)}=\frac{19{,}921.88}{1-\left(\frac{2}{3}\times 0.9795\right)^2(1-0.2369)}=29{,}531 in.^4$$

For compression reinforcement following Design Aid, Deflection 13.6.1:

$$\beta_c=\frac{(n-1)\rho'}{\rho n}=\frac{(9.5-1)0.0048}{0.0061\times 9.5}=0.7041$$

$$K_{i2}=\left[\frac{(c/d)^3}{3}+\rho n\{1-(2\,c/d)+(c/d)^2\}+\rho n\beta_c\left\{(c/d)^2-(2\,c/d)\frac{d'}{d}+\left(\frac{d'}{d}\right)^2\right\}\right]=0.0466$$

$$I_{cr}{}^{-}=K_{i2}b_w d^3=0.0466\times 18\times 27.5^3=17{,}459.14 in.^4$$

The effective moment of inertia using Design Aid, Deflection 13.7.1 is:

$$\frac{M_{cr}}{M_a}=\frac{329.80}{264.06}=1.2490$$

$$\frac{I_{cr}}{I_g}=\frac{17{,}459.14}{84{,}077.64}=0.2077$$

$$I_e{}^{-}=\frac{I_{cr}}{1-\left(\frac{\frac{2}{3}M_{cr}}{M_a}\right)^2\left(1-\frac{I_{cr}}{I_g}\right)}=\frac{17{,}459.14}{1-\left(\frac{2}{3}\times 1.2490\right)^2(1-0.2077)}=38{,}807 in.^4$$

The average effective moment of inertia per 24.2.3.6 is:

$$I_e=\frac{I_e{}^{+}+I_e{}^{-}}{2}=34{,}169 in.^4$$

The initial deflection for total load is:

$$\Delta_t=\frac{w_a\ell^4}{384E_cI_e}=\frac{2.445/12\times(36\times 12)^4}{384\times 3067\times 34{,}169}=0.176 in.<\frac{\ell}{480}=0.9\ in.\ \text{OK}$$

The initial deflection for live load is:

$$\Delta_l = \frac{\frac{0.91}{12} \times (36 \times 12)^4}{384 \times 3067 \times 34{,}169} = 0.065 in. < \frac{\ell}{360} = 1.2\ in.\ \text{OK}$$

The time-dependent deflection calculation for a 5-year sustained load involves Eq. 24.2.4.1.1, assuming 3#7 top rebar at mid-span:

$$\lambda_\Delta = \frac{\xi}{1 + 50\rho'} = \frac{2.0}{1 + 50 \times 0.0036} = 1.69$$

Hence, for sustained dead load and additional live load, the deflection in five years is:

$$\Delta_{@5year} = \lambda_\Delta \Delta_d + \Delta_l = 1.69(0.176 - 0.065) + 0.065 = 0.25\text{in.}$$

Several conclusions may follow these calculations. Primarily, the contemporary design aims for more slender members responsible for larger deflections. This observation is no cause for alarm. With the engineer realizing the existence of these factors, the most satisfactory design for the given conditions is achievable. Varying the depth of the member can control the degree of deflection for both heavy and lightweight concrete members.

A normal weight concrete beam has 10.8% more service moments than a lightweight concrete beam. This value is comparable with the modification factor $\lambda = 0.9$ based on the equilibrium weight. However, the ACI 318 conservatively specifies a modification factor of 0.75 for the rupture stress disregarding the equilibrium weight. Hence, the rupture stress of the normalweight concrete beam is 133% of the one for a lightweight concrete beam. Hence, the cracking-to-service moment ratio is 83% of the same ratio for a lightweight concrete beam. This change increases the calculation of effective moment of inertia by nearly 75%. The additional reinforcing bars required for the heavier load of the normal weight concrete further increase the cracked-section moment of inertia.

Moreover, the modulus of elasticity of normal weight concrete is also 30% more than lightweight concrete. Therefore, the expected deflection of the normal weight concrete beam with an additional 12% service load is 54% less than the lightweight concrete beam. Regardless, the solved example above indicates that the overall results for deflection of lightweight concrete satisfy ACI 318 criteria with nearly 20% fewer reinforcing bars.

In lightweight concrete designs with a constant weight, deflections are reduced more rapidly with an increase in the length of the member than in heavy concrete designs. A similar conclusion applies to the difference between aluminum and steel.

In general, the deflection of beams and slabs with fixed end moments is usually so negligible that the effect of wind and solar heat on measuring devices must be considered.

Presented calculations are also applicable to precast construction.

Deflection of Structural Lightweight Concrete

1 INTRODUCTION

1.1 OBJECTIVES

This Information Sheet intends to present recommendations for structural lightweight aggregate concrete and supplement the requirements presented in the Code. The subject of deflection is treated in general terms in ACI 318 (Building Code Requirements for Structural Concrete). Chapter 24 of the Code has specific requirements, but the applicability to structural lightweight concrete is not always obvious. All information contained herein is intended to apply only to structural lightweight concrete with lightweight aggregate manufactured from shale, clay, or slate by the rotary kiln process.

2 GENERAL DEFLECTION CHARACTERISTICS

2.1 LOAD-DEFLECTION CHARACTERISTICS

It would be well to briefly review the load-deflection characteristics of lightweight concrete flexural members compared to similar ones in normal weight concrete. Figs. 2.1 show load-deflection curves for rectangular beams of the same size and with the same reinforcement made of lightweight and normal weight concrete, designed to carry the same total load. The three lines of OAC are for three different reinforcement percentages, the upper lines representing beams with the greater amount of reinforcing steel. Qualitatively these curves demonstrate that higher percentages of reinforcement decrease deflections of a cracked section.

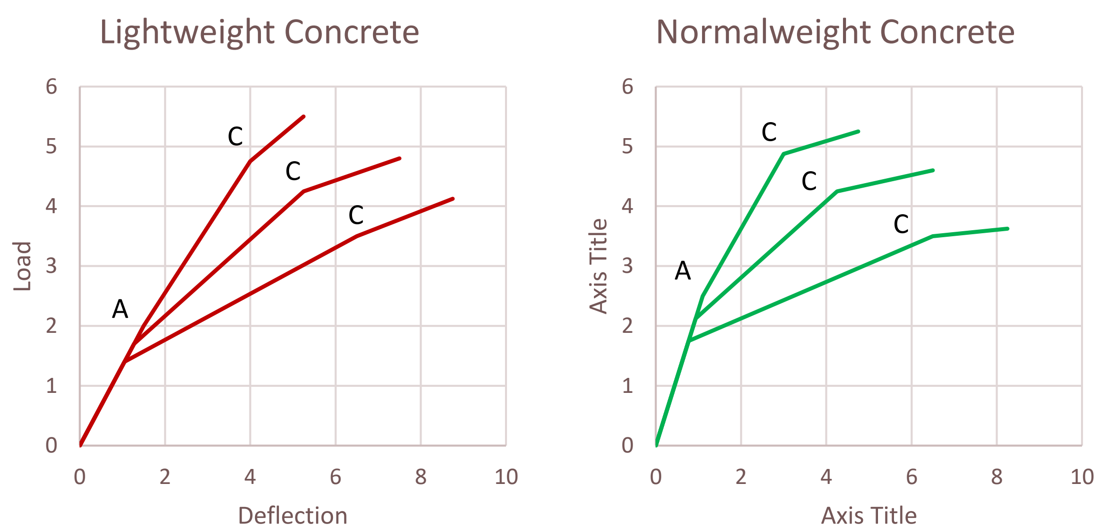

Figure 2.1 Load-Deflection Relationship for Lightweight (left) and Normal weight (right) Beams.

2.2 Cracking Moment

Lightweight concrete and normal weight beams act as homogeneous sections until the cracking stress is reached. At the cracking moment (Points A on Fig. 2.1), the stiffness of the beam is reduced, and the slope of the load-deflection curve changes. The moment required to cause the following relationship gives initial cracking:

$$M_{cr} = \frac{f_{cr} I_g}{y_t} \qquad \text{Eq. 2.1}$$

where f_{cr} is the concrete modulus of rupture, y_t is the distance from the centroid to the extreme tensile fiber, and I_g is the moment of inertia of the gross concrete section neglecting steel.

2.3 The Thickness of Flexural Members

It can be noted from Fig. 2.1 that lightweight members show a lesser reduction in stiffness after cracking than normal weight members. The slopes of the load-deflection curves after cracking are roughly the same for the two types of beams having the same steel area and overall dimensions. The similarity is due to the significant contribution of reinforcing steel to cracked beam stiffness. It is well to remember in reviewing Fig. 2.1 that in a structural system of lightweight aggregate concrete, the dead load of the framing system is approximately 75% of the dead load of a normal weight concrete frame.

Section 9.05 of ACI 318 refers explicitly to flexural members of normal weight concrete. In lightweight structures designed to support the same live load, the dead weight of a normal weight concrete system ranges between 35% and 50% greater.

2.3.1 Normal weight versus Lightweight Member

Considering the deflection of a member as a direct function of total load (TL) and an inverse function of flexural stiffness (EI), a comparative relationship that includes these variables is:

$$\frac{\Delta_{LWC}}{\Delta_{NWC}} \propto \frac{\mathrm{TL}_{LWC}}{\mathrm{TL}_{NWC}} \times \frac{\mathrm{EI}_{NWC}}{\mathrm{EI}_{LWC}} \quad \text{Eq. 2.1}$$

Reducing flexural stiffness to the variable factor in question (t^3) (t = thickness of member; width of section identical), the relationship becomes:

$$\frac{\Delta_{LWC}}{\Delta_{NWC}} \propto \frac{\mathrm{TL}_{LWC}}{\mathrm{TL}_{NWC}} \times \frac{\mathrm{E}_c t^3{}_{NWC}}{\mathrm{E}_c t^3{}_{LWC}} \quad \text{Eq. 2.2}$$

Hence, for obtaining the same deflection, the ratio between thickness values becomes:

$$\frac{\mathrm{t}^3{}_{LWC}}{\mathrm{t}^3{}_{NWC}} \propto \frac{\mathrm{TL}_{LWC}}{\mathrm{TL}_{NWC}} \times \frac{\mathrm{E}_{cNWC}}{\mathrm{E}_{cLWC}} \quad \text{Eq. 2.3}$$

In a structure where the specified live loads are in the normal range of approximately 40 to 80 pounds per square foot, an analysis shows that the total load supported by a normal weight flexural member is between 15% and 25% greater than that of an equal strength lightweight concrete member. Introducing 20% greater as an average for normal weight concrete and concrete weight as the variable factor in determining the modulus of elasticity ($E_{cLWC} = w_c{}^{1.5}33\sqrt{f_c'}$ and $E_{cNWC} = 57{,}000\sqrt{f_c'}$; assume the same strength of concretes), a simplification of the above relationship is:

$$\frac{\mathrm{t}^3{}_{LWC}}{\mathrm{t}^3{}_{NWC}} \cong \frac{1}{1.20} \times \left(\frac{145}{w_{LWC}}\right)^{1.5} \quad \text{Eq. 2.4}$$

$$\frac{t_{LWC}}{\mathrm{t}_{NWC}} \cong \left[\frac{1}{1.20} \times \left(\frac{145}{w_{LWC}}\right)^{1.5}\right]^{1/3} \quad \text{Eq. 2.5}$$

2.3.2 Minimum thickness of non-prestressed concrete members

Table 2-1 and *Table 2-2* reflect values in ACI 318-19 Tables 7.3.1.1 and 9.3.1.1 for minimum thickness of normal weight and lightweight concrete non-prestressed members. Incorporating lightweight concrete in this analysis to these tables with more frequently specified densities of 90, 100, and 110 pcf results in a multiplier per ACI 7.3.1.1.2 and 9.3.1.1.2:

$$max\begin{cases} 1.65 - 0.005w_c \\ 1.09 \end{cases}$$

Table 2-1 Minimum thickness for non-prestressed one-way slabs unless deflections are computed.

Concrete Density	Simply supported	One end continuous	Both ends continuous	Cantilever
Normal weight ($w_c = 145\ pcf$)	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Lightweight ($w_c = 110\ pcf$)	$\ell/18.2$	$\ell/21.8$	$\ell/25.5$	$\ell/9.1$
Lightweight ($w_c = 100\ pcf$)	$\ell/17.4$	$\ell/20.9$	$\ell/24.3$	$\ell/8.7$
Lightweight ($w_c = 90\ pcf$)	$\ell/16.7$	$\ell/20$	$\ell/23.3$	$\ell/8.3$

After ACI 318-19 Tables 7.3.1.1 and 7.3.1.1.2

Table 2-2 Minimum thickness for non-prestressed beams unless deflections are computed.

Members	Simply supported	One end continuous	Both ends continuous	Cantilever
Normal weight ($w_c = 145\ pcf$)	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$
Lightweight ($w_c = 90\ pcf$)	$\ell/14.5$	$\ell/16.8$	$\ell/19.1$	$\ell/7.3$
Lightweight ($w_c = 90\ pcf$)	$\ell/13.9$	$\ell/16.1$	$\ell/18.3$	$\ell/7.0$
Lightweight ($w_c = 90\ pcf$)	$\ell/13.3$	$\ell/15.4$	$\ell/17.5$	$\ell/6.7$

After ACI 318-19 Tables 9.3.1.1 and 9.3.1.1.2

The discretionary use of the above span-to-depth ratios can save the designer immeasurable time in establishing the basic dimensions they can use for construction depths when planning a project. But these values are no substitute for the detailed analysis of a concrete member, whether lightweight or normal weight concrete, for determining the elastic deflection or the theoretical deflection with a reasonable degree of accuracy. It is, therefore, recommendable that the above tables serve only within the bounds of the permission granted by the ACI Code. Note that the above tables do not apply to tensioned concrete.

3 Reinforced Concrete

3.1 Short-term deflection

The determination of the deflection of any flexural member may employ integration, moment area methods, or numerical procedures involving angle changes. The deflection relationships take some form of these equations:

$$\Delta = \gamma \frac{M\ell^2}{E_c I_e} = \gamma \phi \ell^2 \qquad \text{Eq. 3.1}$$

where γ is a constant related to the change in elastic curvature, ϕ distribution along the member, determined by the loading and support conditions, E_c is the modulus of elasticity of concrete, I_e is the effective moment of inertia, ϕ is the angle of rotation per unit length as follows:

$$\phi = \varepsilon / h \qquad \text{Eq. 3.2}$$

where ε is the total strain, the algebraic sum of the top and bottom fiber strains, and *h* is the depth of the flexural member.

3.2 Effective Moment of Inertia

Section 24.2.3.6 of ACI 318 suggests the average value of the effective moment of inertia in the positive and negative regions for continuous spans. Whether the gross section transformed cracked section, or some average value is used for substitution in the above equations, any constant value for the *EI* term describes a straight-line load-deflection relationship such as line OB in Figs. 2.1. The use of average values of *EI* may well be accurate enough for most reinforced concrete flexural members since long-term effects are not accurately known. More accurate computations of elastic deflections are available if desired by using the gross section where the actual bending moment is less than the cracking moment and the cracked section where the actual moment is higher than the cracking moment.

3.3 Cracked Section

It is possible to obtain accurate results for deflections in the uncracked range because both *E* and *I* are constant and determined with some degree of precision.

In the cracked range, the following equation determines *EI*:

$$EI_{cr} = E_c \frac{bc^3}{3} + E_s A_s (d - c)^2 \qquad \text{Eq. 3.2}$$

where *b* is the width, *d* is the effective depth of the tensile steel reinforcement, A_s is the tensile steel reinforcement, *c* is the depth of the neutral axis to the effective depth of the beam, E_c is the concrete modulus of elasticity, and E_s is the steel modulus of elasticity.

This information sheet includes a typical deflection calculation for a simple rectangular beam, a continuous span, and a two-way system considering long-term effects.

4 Long-term Deflection

4.1 Time-Dependent Effects

The preceding discussion has dealt with elastic deflection. In addition to the deflection caused by elastic strains in the tensile and compressive zones, there is further deflection in reinforced concrete beams due to creep and shrinkage strains.

4.2 Shrinkage and Creep Strains

Shrinkage strains increase with time and have a root cause in drying, which is assumed to be independent of the tensile or compressive stress. Without reinforcement, the shrinkage strain would be uniform throughout the depth in a plain concrete beam. This observation is also accurate for a beam with symmetrical reinforcement. However, if the beam only has steel in the bottom, the top of the beam shrinks more than the bottom, causing additional deflection due to the shortening of the top surface relative to the bottom. Fig. 4.1 illustrates the strain regime for a cracked beam.

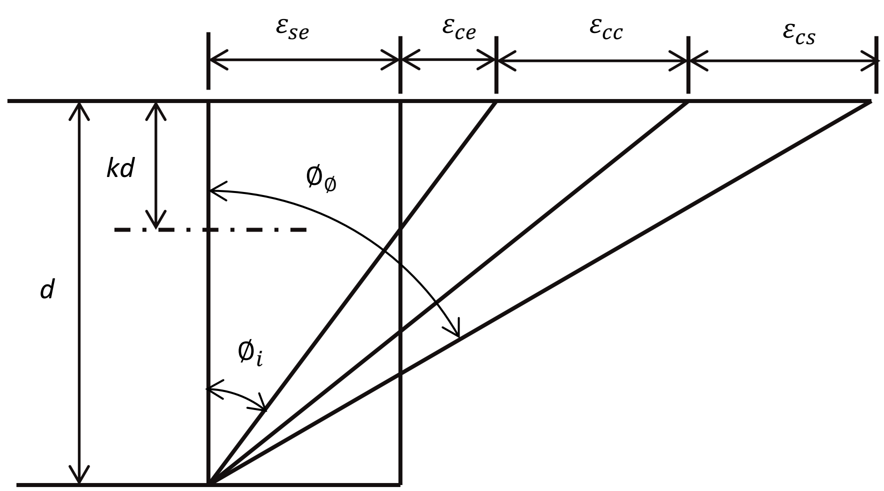

Figure 4.1 Strain relationship for a cracked beam (ε_{se} = elastic strain in steel, ε_{ce} = elastic strain in extreme concrete fiber, ε_{cc} = creep strain in extreme concrete fiber, ε_{cs} = shrinkage strain in extreme concrete fiber relative to reinforcing steel)

The angle ϕ_i represents the elastic curvature of a cracked reinforced concrete beam, which is:

$$\phi = (\varepsilon_{se} + \varepsilon_{ce})/d \qquad \text{Eq. 4.1}$$

The elastic deflection of a uniformly loaded beam follows the second expression of Equation 3.1 with $\gamma = 5/48$. We can consider the creep strain in reinforced concrete only in the compression zone since reinforcement controls strains in the tension zone.

By applying suitable values to the strains given in Fig. 4.1, it is possible to develop a factor that can be applied to the elastic deflection due to sustained loading to obtain long-term deflection. This factor or multiplier is the ratio ϕ_ϕ/ϕ_i.

The multipliers in Table 4-1 for cracked beams rely on the following relationship:

$$\varepsilon_{ce} = \frac{f_c}{E_c} \quad \text{Eq. 4.2}$$

$$\varepsilon_{se} = \varepsilon_{ce} d \frac{1-k}{kd} \quad \text{Eq. 4.3}$$

$$\varepsilon_{cc} = m\varepsilon_{ce} \quad \text{Eq. 4.4}$$

where *m* is a function of A'_s, the compressive reinforcing steel ratio:

$$m = \begin{cases} 0 \; for \; A'_s = 0 \\ 2.5 \; for \; A'_s = 0.5A_s \\ 2.25 \; for \; A'_s = A_s \end{cases} \quad \text{Eq. 4.5}$$

$$\varepsilon_{cs} = \begin{cases} 0.042(\rho - \rho') \text{ for lightweight concrete} \\ 0.035(\rho - \rho') \text{ for normalweight concrete} \end{cases} \quad \text{Eq. 4.6}$$

where, $\rho = \frac{A_s}{bd}$ and $\rho' = \frac{A'_s}{bd}$ are compressive and tensile reinforcement ratios, respectively.

The term (*m*) relating creep strain to elastic strain is not known as a specific value for either normal weight or lightweight concrete and varies with the mix. However, the factor (*m*) is not significantly different for lightweight structural concrete than for normal weight.

4.3 Time-Dependent Deflections

Table 4.1 gives deflection multipliers for lightweight concrete beams and slabs. The multipliers are to be applied to the elastic deflections due to sustained loading only to obtain additional long-term deflections. Table 4-1 presents multipliers for early ages indicating how deflection increases with time.

*Table 4-1 Additional Long-Term Deflection Multipliers for Lightweight Concrete**

Duration under Sustained Load	$A'_s = 0$	$A'_s = 0.5A_s$	$A'_s = A_s$
1 Month	0.6	0.4	0.3
3 Months	0.9	0.7	0.5
6 Months	1.2	0.9	0.6

1 Year	1.5	1.1	0.7
5 Years	2.1	1.2	0.8

* These multipliers are to be applied to the elastic deflection due to sustained loading only (usually dead load) to obtain additional long-term deflections.

Table 4.1 shows that a substantial amount of plastic deflection occurs in the first three months. Table 4.1 also emphasizes that long-term deflections due to shrinkage and creep can be reduced substantially by the addition of compressive reinforcement.

Section 24.2.4 of ACI 318 gives multipliers for additional long-term deflections. Since these Code values follow a 5-year duration for normal weight concrete, a comparison with Table 4.1 shows that the same values are also valid for structural lightweight concrete. Table 4.2 is an expanded version of ACI 318 Table 24.2.4.1.3 for time-dependent amplification of deflections.

Table 4-2 Time-dependent factor for sustained loads

Duration under Sustained Load	$\rho' = 0$	$\rho' = 0.01$	$\rho' = 0.02$
3 Month	1.0	0.67	0.5
6 Months	1.2	0.80	0.6
12 Months	1.4	0.93	0.7
5 Years	2.0	1.33	1.0

5 Continuous Beams and Slabs

Computation of the deflection of continuous beams and slabs follows these relationships:

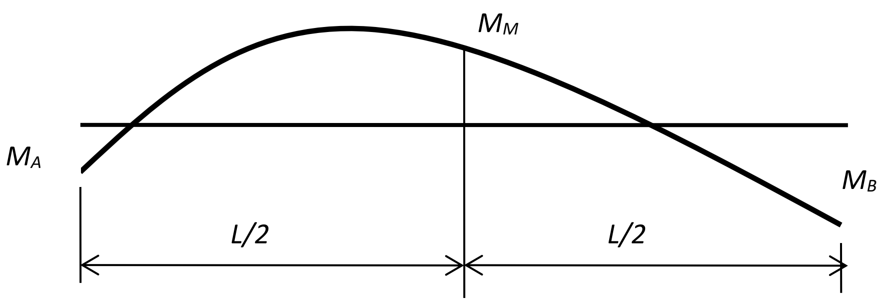

Figure 5.1 Moment distribution of a continuous beam.

The deflection of an uncracked section is:

$$\Delta_M = 5L^2[M_M - 0.1(M_A + M_B)]/48E_cI_c \quad \text{Eq. 5.1}$$

The deflection of a cracked section is:

$$\Delta_M = 5L^2[M_M - 0.1(M_A + M_B)]/48E_sA_sd^2R \quad \text{Eq. 5.2}$$

where, M_A and M_B are negative moments at support, and M_M is the net positive moment at midspan, but not necessarily the maximum positive moment.

6 Prestressed Concrete

6.1 Short-Term

Determination of the short-term camber and deflection of prestressed concrete members employs the usual deflection calculation tools, such as the conjugate beam or moment area methods. The effect of the prestressing force involves plotting the f_{pe} moment across the length of the beam. For a beam having straight tendons, the f_{pe} plot is a rectangle. Beams having deflected strands or curved tendons have *M/EI* diagrams that follow the eccentricity of the steel across the length. Since the moment caused by the prestress f_{pe} is almost always of opposite sign to the moments caused by dead and live loads, the plots of *M/EI* diagrams for the prestress and load conditions are on opposite sides of a conjugate beam. Most precast pretensioned members have a simple span design; therefore, the f_{pe} moment causes an upward deflection or camber, and the dead and live loads create a downward deflection, as in the case of reinforced concrete.

6.2 Long-term

Determining the long-term camber or deflection of a prestressed member requires considering several factors that were not present in the case of reinforced concrete. First, the amount of creep that takes place in prestressed concrete is less than for reinforced concrete because the concrete is of higher compressive strength at the loading time. Creep tends to increase dead-load deflection, as in the case of reinforced concrete, but at the same time, it increases the camber caused by prestressing. If a beam had a camber from prestressing that was precisely equal to the dead-load deflection, the net effect would be zero deflection, except that the prestressing force relaxes with time. Because of this, the net effect would cause the beam to deflect slightly.

7 CAMBER

7.1 PRESTRESS LOSS

ACI 318 Section 23.3.2.6 outlines the factors contributing to camber, deflection, and prestress loss. The only difference between lightweight and normal weight concrete is elastic shortening and shrinkage since the effect of creep has been assumed to be the same. The steel stress losses in a lightweight pre-tensioned beam are about 46,000 psi when the average initial compression stress is 0.25f'c (release at 4000 psi) and about 37,000 psi at an average initial compressive stress of 0.12f'c. Under similar conditions, the loss in normal weight concrete would be 38,500 psi and 32,000 psi. Therefore, the steel losses in lightweight are 16 to 20 percent more than in normal weight of comparable strength.

7.2 AGGREGATE SOURCE

It would be necessary at this time to impress upon the designers to know the source of the local aggregate and to rely upon the producer to provide them with the characteristics of the aggregate under consideration. Since shrinkage and creep characteristics for lightweight aggregates, like for natural normal weight concrete aggregates, vary in geographical locations.

7.3 LONG-TERM EFFECTS

Computing short and long-term camber (or deflection) for prestressed concrete members follows superimposing the effects of prestress on typical live and sustained load effects. Long-term effects result from multiplying the net camber or deflection under sustained loading by the appropriate factor. Theoretically, shrinkage should not affect the camber or deflection of prestressed concrete where typical reinforcement is not present. Shrinkage deflections in regularly reinforced concrete occur because the reinforcement does not shorten in the tension zone. This observation is inaccurate in prestressed concrete where the tendons shorten with the concrete. Hence no angle changes should occur due to shrinkage itself. This change doesn't happen for creep since it is a function of the compressive stress in the concrete at any point. Thus, if the computation of deflections (or camber) relies on the reduced final stress in the tendons, the additional deflections (or camber) due to sustained loading should only be adjusted for creep effects. Creep effects alone can sometimes double the deflection (or conversely camber) under sustained loading.

8 PONDING

There have been cases, with all kinds of building materials, where a roof collapsed due to the ponding of water. The problem can occur if the roof possesses a drape under its self-weight and conditions are such that water cannot leave the roof during heavy rain. Under these conditions, the roof does not collapse if the member has sufficient stiffness, so an additional inch of water does not induce more than an inch of deflection. Lightweight reinforced concrete and prestressed concrete roof systems have sufficient stiffness if the cracked I is greater than the minimum cracked moment of inertia per unit linear feet of the width of $45 \times 10^{-6} L^4$, where *L* is the span in feet. This value is based on *E*=2,000 ksi. This value decreases proportionately as *E* increases; that is, for concrete with E = 4,000 ksi, the minimum *I* is $22.5 \times 10^{-6} L^4$.

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